1 a) onjuist tegenvoorbeeld → $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ is convergent

dus stel $a_n = (-1)^n \frac{1}{n}$

Maar de reeks $\sum_{n=1}^{\infty} (-1)^n a_n = \sum_{n=1}^{\infty} (-1)^n (-1)^n \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ is divergent

dus als $\sum_{n=1}^{\infty} a_n$ convergent is, hoeft niet te gelden dat $\sum_{n=1}^{\infty} (-1)^n a_n$ convergent is

b) juist, bewijs → $0 \le |a_n| + a_n \le 2|a_n|$ want $|a_n| = a_n$ of $|a_n| = -a_n$

$\sum_{n=1}^{\infty} |a_n|$ is convergent

$(-1)^n a_n$ heb ik a_n genoemd

dus $2\sum_{n=1}^{\infty} |a_n|$ is ook convergeert want gewoon keer een constante

Dus $\sum_{n=1}^{\infty} (|a_n| + a_n)$ is ook convergent (volgens de comparison test, want $|a_n| + a_n$ is altijd kleiner of gelijk aan $2|a_n|$)

$\sum_{n=1}^{\infty} (|a_n| + a_n) - \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} a_n$

Dus $\sum_{n=1}^{\infty} a_n$ is ook convergent want het is een verschil van twee convergente rijen, en dan is hij convergent

nice

c) juist, bewijs ~~$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n a_n$~~

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} a_n$ als a_n alleen positieve waardes heeft, dus dan is $\sum_{n=1}^{\infty} a_n$ sowieso absolunt convergent. (want $\sum_{n=1}^{\infty} a_n$ is convergent)

~~Als a_n alleen uit negatieve ... waardes heeft~~. Als a_n alternerend is, dan $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} (-1)^n a_n$, en $\sum_{n=1}^{\infty} (-1)^n a_n$ is convergent dus $\sum_{n=1}^{\infty} |a_n|$ is convergent dus $\sum_{n=1}^{\infty} a_n$ is absolunt convergent. wordt vervolgd....

wordt vervolgd

2 $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2^n \ln n}$ $a_n = \frac{(-1)^n x^n}{2^n \ln n}$ $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{2^{n+1} \ln(n+1)} \cdot \frac{2^n \ln n}{x^n}\right|$

$= \frac{|x| \ln n}{2 \ln(n+1)}$

$= \frac{1}{2}|x| \cdot \frac{\ln n}{\ln(n+1)}$

$f(x) = \frac{\ln x}{\ln(x+1)}$

$\lim_{x\to\infty} \frac{\ln x}{\ln(x+1)} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x+1}} = \lim_{x\to\infty} \frac{x+1}{x}$

L'hôspital

$= \lim_{x\to\infty} \frac{1+\frac{1}{x}}{1} = \frac{1+0}{1} = 1$

$\lim_{n\to\infty} \left(\frac{1}{2}|x| \cdot \frac{\ln n}{\ln(n+1)}\right) = \frac{1}{2}|x|$

Dus volgens de Ratio test convergeert de reeks absolunt als $\frac{1}{2}|x| < 1$, dus $|x| < 2$, dus $-2 < x < 2$

Als $x = -2$ $\sum_{n=1}^{\infty} \frac{(-1)^n (-2)^n}{2^n \ln n} = \sum_{n=1}^{\infty} \frac{2^n}{2^n \ln n} = \sum_{n=1}^{\infty} \frac{1}{\ln n}$

$\ln n < n$ voor $n = 1,2,3$

dus $\frac{1}{\ln n} > \frac{1}{n}$ voor $n = 1,2,3$

omdat $\sum_{n=1}^{\infty} \frac{1}{n}$ divergent is, is dus ook $\sum_{n=1}^{\infty} \frac{1}{\ln n}$ divergent (comparison test)

Als $x=2$ $\quad \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{2^n \ln n}$, een alternerende reeks die convergeert

als geldt ① $\lim_{n\to\infty} b_n = 0$ ② $b_{n+1} \leq b_n$ vanaf een bepaalde n

① $\lim_{n\to\infty} \frac{1}{\ln n} = 0$ (want $\ln n \to \infty$ als $n \to \infty$) met $b_n = \frac{2^n}{2^n \ln n} = \frac{1}{\ln n}$

② stel $f(x) = \frac{1}{\ln x}$ $\quad f'(x) = -\frac{1}{(\ln x)^2} \cdot \frac{1}{x} = -\frac{1}{x(\ln x)^2} < 0$ als $x > 0$

dus vanaf $x=1$ is de afgeleide kleiner dan 0 dus dan daalt de grafiek

dus dan geldt $b_{n+1} \leq b_n$ (~~...~~ voor $n \geq 1$)

Dus voor $x=2$ is de reeks convergent.

Dus de reeks is convergent op het interval $[-2, 2]$, ~~...~~

Dus voor $x=2$ is de reeks convergent, maar niet absoluut convergent

Dus voor $x=2$ is de reeks conditionally convergent.

③ a) $f(x,y) = \frac{x \sin y}{x^2+y^2}$

via de x-as (dus $y=0$) naar $(0,0)$

$f(x,0) = \frac{x \cdot 0}{x^2} = 0$

via $y=x$ naar $(0,0)$

$f(x,x) = \frac{x \sin x}{2x^2}$ $\quad \lim_{x\to 0} \frac{x \sin x}{2x^2} = \lim_{x\to 0} \frac{\sin x}{2x}$

$\overset{\text{l'hôspital}}{=} \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$ 8

Dus verschillende paden geven verschillende limieten dus $\lim_{(x,y)\to(0,0)} f(x,y)$ bestaat niet

b) $f(x,y) = \frac{x^2 \sin y}{x^2+y^2}$ $\quad \forall \varepsilon > 0 \ \exists \delta > 0$ zodanig dat

als $\sqrt{(x-0)^2+(y-0)^2} < \delta \to |f(x,y) - 0| < \varepsilon$

dus $\sqrt{x^2+y^2} < \delta \to \left|\frac{x^2 \sin y}{x^2+y^2}\right| < \varepsilon$

want $|\sin y| \leq |y|$

$\frac{x^2|\sin y|}{x^2+y^2} \leq \frac{x^2|y|}{x^2+y^2} < \frac{x^2 \cdot |y|}{x^2} = |y| = \sqrt{y^2} < \sqrt{y^2+x^2} < \delta$ (want $x^2 > 0$)

want $y^2 > 0$ dus $x^2+y^2 > x^2$ dus $\frac{1}{x^2+y^2} < \frac{1}{x^2}$

~~...~~

Dus Als $\delta = \varepsilon$

dan $\frac{x^2|\sin y|}{x^2+y^2} < \varepsilon$ als $\sqrt{x^2+y^2} < \delta$

Dus dan geldt dat $\sqrt{(x-0)^2+(y-0)^2} < \delta \to |f(x,y)-0| < \varepsilon$

~~...~~

dus $|f(x,y)-0| = \frac{x^2|\sin y|}{x^2+y^2} \leq \frac{x^2|y|}{x^2+y^2} < \frac{x^2|y|}{x^2} = |y| = \sqrt{y^2} < \sqrt{y^2+x^2} < \varepsilon$ (want $\varepsilon = \delta$)

c) $f(x,y) = \frac{x^n \sin y}{x^2+y^2}$ ~~via de lijn x~~

via de x-as is ~~f(x,y) altijd~~. $\lim_{(x,y)\to(0,0)} f(x,y)$ altijd 0. (want $\sin 0 = 0$)

via $x = y$

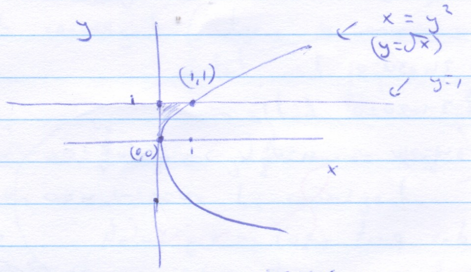
$$f(x,x) = \frac{x^n \sin x}{2x^2} \quad \text{met } n > 2$$

$$= \frac{x^{n-2}\sin x}{2}, \quad \lim_{x\to 0} \frac{x^{n-2}\sin x}{2} = 0$$ (met $n>2$ dus $n-2 > 0$)

dus ik denk dat de limiet wel bestaat (en dan is hij dus 0)

8

4) $\iint_E e^{y^3}\, dA$



$$\int_0^1\int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx$$

$$= \int_0^1\int_{\sqrt{x}}^1 e^{y} \cdot e^{y^2}\, dy\, dx$$

$$= \int_0^1 \left(e^y e^{y^2}\Big|_{\sqrt{x}}^1 - \int_{\sqrt{x}}^1 e^y \cdot e^{y^2} \cdot 2y\, dy \right) dx$$

$$= \int_0^1 \left(e \cdot e^{x\sqrt{x}} - \ldots\ e^{y^3} \cdot y^2 \Big|_{\sqrt{x}}^1 + \int_{\sqrt{x}}^1 3y^2 e^{y^3} \cdot y^2\, dy \right) dx$$

$0 \le x \le 1$

$\sqrt{x} \le y \le 1$

$0 \le x \le y^2$

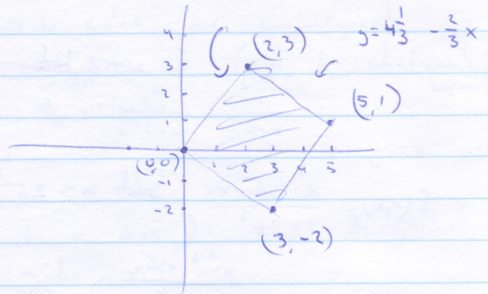
$0 \le y \le 1$

$$\int_0^1\int_0^{y^2} e^{y^3}\, dx\, dy = \int_0^1 x e^{y^3}\Big|_0^{y^2} dy = \int_0^1 y^2 e^{y^3}\, dy$$

$$= \frac{1}{3} e^{y^3}\Big|_0^1 = \frac{1}{3}e - \frac{1}{3} = \frac{1}{3}(e-1)$$

8

5 $\iint_R (x+y)\,dA$



$x = 2u + 3v$

$y = 3u - 2v$

jacobiaan: $\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -4 - 9 = -13$

$(x+y) = 2u + 3v + 3u - 2v = (5u + v)$

$2u + 3v = 0$

$3u - 2v = 0$

$3u = 2v \rightarrow u = \frac{2}{3}v \rightarrow \frac{4}{3}v + \frac{3v}{1} = 0$

$v = 0 \rightarrow u = 0$

dus punt (0,0) in xy-vlak komt overeen met (0,0) in uv-vlak

en ik heb weer eens veel te moeilijk gedaan want je ziet eigenlijk meteen wel dat het om de punten (0,0) (1,0) (0,1) (1,1) gaat.

$2u + 3v = 2 \qquad u = \frac{2-3v}{2}$

$3u - 2v = 3$

$\frac{3(2-3v)}{2} - 2v = 3$

$\frac{6 - 9v}{2} - 2v = 3$

$3 - \frac{9}{2}v - 2v = 3$

$6\frac{1}{2}v = 0 \rightarrow v = 0$

$2u = 2 \rightarrow u = 1$

dus (2,3) komt overeen met (1,0) in uv-vlak

$2u + 3v = 5$

$3u - 2v = 1$

$u = \frac{5-3v}{2}, \quad \frac{15 - 9v}{2} - 2v = 1$

$7{,}5 - \frac{9}{2}v - 2v = 1$

$6\frac{1}{2}v = 6{,}5 \rightarrow v = 1$

$2u + 3 = 5$

$2u = 5 - 3 = 2 \rightarrow u = 1$

dus punt (5,1) in xy-vlak is punt (1,1) in uv-vlak

$2u + 3v = 3$

$3u - 2v = -2$

$u = \frac{3-3v}{2}$

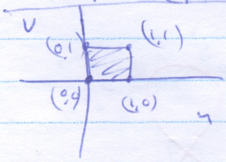
$\frac{9 - 9v}{2} - 2v = -2$

$4{,}5 - 6\frac{1}{2}v = -2$

$6\frac{1}{2}v = 6\frac{1}{2} \rightarrow v = 1$

$2u + 3 = 3 \rightarrow u = 0$

dus punt (0,1)



$0 \le u \le 1$

$0 \le v \le 1$

dus $13 \int_0^1 \int_0^1 (5u + v)\,du\,dv = 13 \int_0^1 \frac{5}{2}u^2 + uv \Big|_0^1 dv = 13 \int_0^1 \left(\frac{5}{2} + v\right) dv$

$= 13 \left[\frac{5}{2}v + \frac{1}{2}v^2 \right]_0^1 = 13 \left(\frac{5}{2} + \frac{1}{2} \right) = 13 \cdot 3 = 39$

8

(6) a) $y''(x) - 4y'(x) + 5y(x) = 0$

karakt. vgl $\longrightarrow$ $r^2 - 4r + 5 = 0$

$(r-2)^2 + 1 = 0 \rightarrow r = \pm\sqrt{-1} + 2$

$= \pm i + 2$

7 dus $y(x) = e^{2x}\left(c_1 \cos x + c_2 \sin x\right)$

b) gok $\left.\begin{array}{l} y_p = Ae^{-x} \\ y_p' = -Ae^{-x} \\ y_p'' = Ae^{-x} \end{array}\right\}$ invullen $\rightarrow$ $Ae^{-x} + 4Ae^{-x} + 5Ae^{-x} = 20e^{-x}$

$e^{-x}(A + 4A + 5A) = 20e^{-x}$

$10A = 20 \rightarrow A = 2$

5 Dus $y_p = 2e^{-x}$

dus $y(x) = e^{2x}\left(c_1 \cos x + c_2 \sin x\right) + 2e^{-x}$ (homogene oplossing van vraag a erbij)

c) $y(x) = e^{2x}\left(c_1 \cos x + c_2 \sin x\right) + 2e^{-x}$

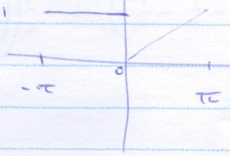
$y(0) = c_1 + 2 = 2 \rightarrow c_1 = 0$

$y'(x) = e^{2x}\left(-c_1 \sin x + c_2 \cos x\right) + 2e^{2x}\left(c_1 \cos x + c_2 \sin x\right) - 2e^{-x}$

2 $y'(0) = c_2 + 2(c_1) - 2 = c_2 - 2 = -2 \rightarrow c_2 = 0$

Dus $y(x) = 2e^{-x}$

(7) $f(x) = \begin{cases} 1 & -\pi \leq x \leq 0 \\ x & 0 < x \leq \pi \end{cases}$ $\left(f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)\right)$



$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{0} 1\,dx + \frac{1}{2\pi}\int_{0}^{\pi} x\,dx$

$= \frac{1}{2\pi}\left[x\right]_{-\pi}^{0} + \frac{1}{2\pi}\left[\frac{1}{2}x^2\right]_{0}^{\pi} = \frac{1}{2\pi}\cdot\pi + \frac{1}{2\pi}\cdot\frac{\pi^2}{2}$

$= \frac{1}{2} + \frac{1}{4}\pi$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{0}\cos(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\sin(nx)\right]_{-\pi}^{0} + \frac{1}{\pi}\left(\frac{x}{n}\sin(nx)\Big|_0^{\pi} - \int_0^{\pi}\frac{1}{n}\sin(nx)\,dx\right)$$

$$= \frac{1}{\pi}(0-0) + \frac{1}{\pi}\left((0-0) + \frac{1}{n^2}\cos(nx)\Big|_0^{\pi}\right)$$

als $\cancel{n = \text{even}}$

$$= \frac{1}{\pi}\left(\frac{1}{n^2}\cos(nx)\Big|_0^{\pi}\right) = \frac{1}{\pi}\left(\frac{1}{n^2} - \frac{1}{n^2}\right) = 0$$

als $n = \text{oneven}$ $\rightarrow$ $\frac{1}{\pi}\left(\frac{1}{n^2}\cos(nx)\Big|_0^{\pi}\right) = \frac{1}{\pi}\left(-\frac{1}{n^2} - \frac{1}{n^2}\right) = -\frac{2}{\pi n^2}$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0}\sin(nx)\,dx + \frac{1}{\pi}\int_0^{\pi} x\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left(-\frac{1}{n}\cos(nx)\Big|_{-\pi}^{0}\right) + \frac{1}{\pi}\left(-\frac{x}{n}\cos(nx)\Big|_0^{\pi} + \int_0^{\pi}\frac{1}{n}\cos(nx)\,dx\right)$$

als $n = \text{even}$

$$= \frac{1}{\pi}\left(-\frac{1}{n} + \frac{1}{n}\right) + \frac{1}{\pi}\left(\left(-\frac{\pi}{n} + \frac{\pi}{n}\right) + \frac{1}{n^2}\sin(nx)\Big|_0^{\pi}\right) = 0$$

als $n = \text{oneven}$

$$\frac{1}{\pi}\left(-\frac{1}{n}\cos(nx)\Big|_{-\pi}^{0}\right) + \frac{1}{\pi}\left(-\frac{x}{n}\cos(nx)\Big|_0^{\pi} + \int_0^{\pi}\frac{1}{n}\cos(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(-\frac{1}{n} - \frac{1}{n}\right) + \frac{1}{\pi}\left(\left(\frac{\pi}{n} + \frac{\pi}{n}\right) + \frac{1}{n^2}\sin(nx)\Big|_0^{\pi}\right) = -\frac{2}{\pi n} + \frac{1}{\pi}\left(\frac{2\pi}{n} + 0\right)$$

RFT

$$= -\frac{2}{\pi n} + \frac{2}{n} = \frac{2}{n}\left(1 - \frac{1}{\pi}\right)$$

Dus $f(x) = \frac{1}{2} + \frac{1}{4}\pi + \sum_{n=1}^{\infty} -\frac{2}{\pi n^2}\cos(nx) + \sum \frac{2}{n}\left(1 - \frac{1}{\pi}\right)\sin(nx)$ met $n = \text{oneven}$

$$= \frac{1}{2} + \frac{1}{4}\pi + \sum_{n=1}^{\infty} -\frac{2}{\pi(2n-1)^2}\cos\big((2n-1)x\big) + \frac{2}{2n-1}\left(1 - \frac{1}{\pi}\right)\sin\big((2n-1)x\big)$$

op interval $[-\pi, \pi]$

① c. Als a_n alleen maar negatieve waardes heeft,

bv $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$ en $a_1, a_2, \dots$ is negatief

~~dan~~ $\cancel{\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - \dots}$

~~dat is een alternerende reeks, en deze is convergent~~

stel $b_n = |a_n|$

~~dan~~ $\cancel{\sum_{n=1}^{\infty} a_n = -b_1 - b_2 - b_3 - \dots}$

$\cancel{\sum_{n=1}^{\infty} (-1)^n b_n = b_1 - b_2 + b_3 - b_4 \dots}$

$\sum_{n=1}^{\infty} a_n = (-1) \sum_{n=1}^{\infty} b_n$

~~convergent~~ En $\sum_{n=1}^{\infty} a_n$ is convergent,

dus $(-1) \sum_{n=1}^{\infty} b_n$ is convergent

dus ook $\sum_{n=1}^{\infty} b_n$ is convergent (want -1 is gewoon een constante)

dus $\sum_{n=1}^{\infty} |a_n|$ is convergent, dus de

reeks $\sum_{n=1}^{\infty} a_n$ is absoluut convergent